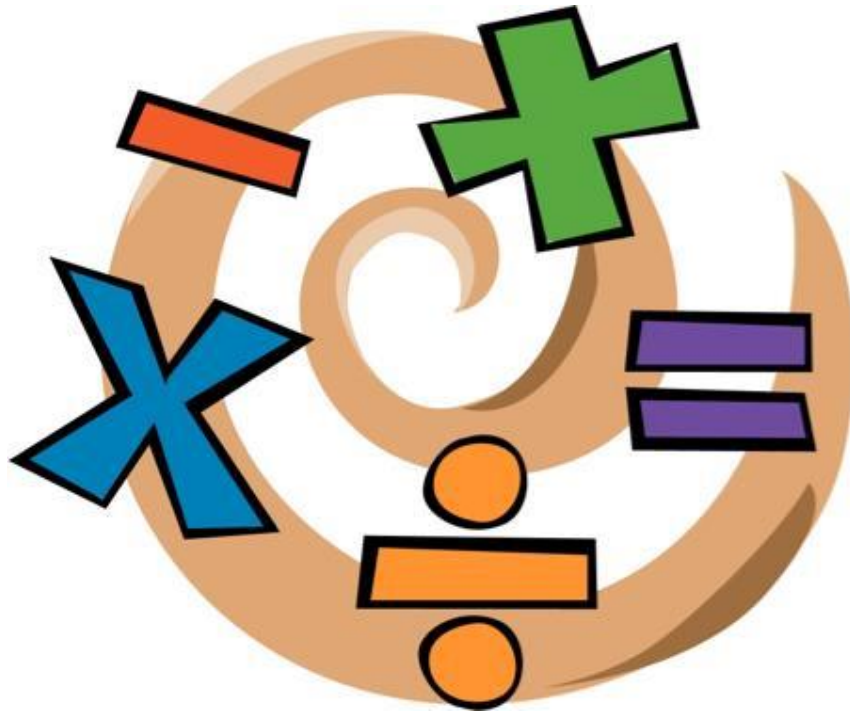


STARTING OUT



WITH NUMBER

A STEP-BY-STEP GUIDE TO MODERN METHODS
USED IN U.K. PRIMARY MATHEMATICS.

By R. Tutchter

2014

Preface

This booklet is the original, unpublished, independent work of the author, R. Tutchter. It is completed in order to support work undertaken in a PhD in Mathematics Education at the Graduate School of Education, University of Bristol. It is also compiled in response to questionnaires returned and feedback received from parents with regards to concerns when supporting their children with homework.

This booklet also supports the workshops delivered by the author.

Special thanks go to LB, AC, JP and CP.

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Once secure with an operation, children will progress onto decimal calculations. This is expected by the end of KS2.

Visual ADDITION

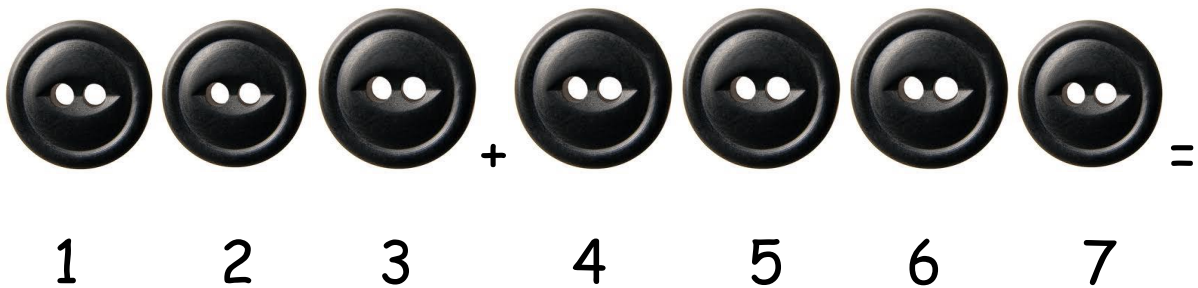
$$3 + 4 = ?$$

Find something you can easily pick up - like buttons.

Pick up three buttons and put them in a row.

Then take four more buttons and place them next to them.

Then by touching them going along the row and counting out loud, add them together.



ADDITION using a number line.

$11+22= ?$

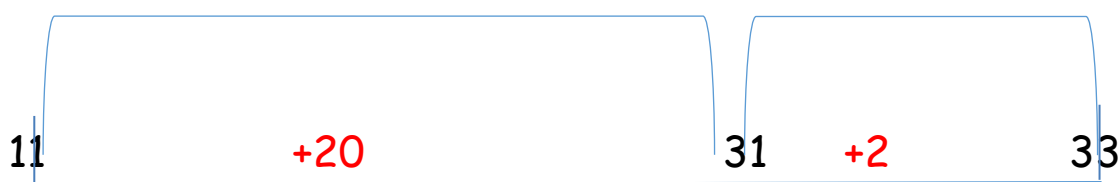
1) Draw a line:



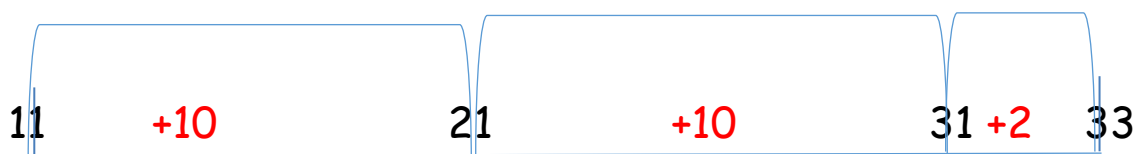
2) Put the first number at the start of the line.



3) Make steps on the number line according to the number you are adding.



On the above example, 22 has been partitioned into 20 and 2 to help simplify the addition. The size of the steps made are the choice of the person doing the mathematics and as long as in the end the steps add to 22 that is fine. Below is an example where the steps are smaller $10 + 10 + 2$.



ADDITION : Partitioning

$$11 + 22 =$$

First partition the numbers into **tens** and **units**.

$$11 = 10 \text{ and } 1$$

$$22 = 20 \text{ and } 2$$

Then add the **tens** together and then the **units**.

$$10 + 20 = 30 \qquad 1 + 2 = 3$$

Add the answers together $30 + 3 = 33$

.....

OR... in a more visual way this could be done as below.

$$\begin{array}{ccc} 11 & + & 22 \\ & \text{becomes} & \\ 10 & 1 + 20 & 2 \\ | & \text{---} & | \\ 30 & & 3 \end{array}$$

.....

Another way to partition with bigger numbers is...

$$672 + 321 =$$

$$600 + 70 + 2$$

$$\underline{300 + 20 + 1}$$

$$900 + 90 + 3 = 993$$

ADDITION : Standard Columns

$11 + 22 =$

KEY:

U = units

T = tens

Step one... write the sum in columns.

$$\begin{array}{r} \text{TU} \\ 11 \\ \underline{22+} \end{array}$$

Step two... add the units column

$$\begin{array}{r} \text{TU} \\ 11 \\ \underline{22+} \\ \underline{3} \end{array}$$

Step three... add the tens column

$$\begin{array}{r} \text{TU} \\ 11 \\ \underline{22+} \\ \underline{33} \end{array}$$

Column Addition with a carry over...

$12 + 69 =$

KEY:

U = units

T = tens

Step 1: write the sum.

$$\begin{array}{r} \text{T} \quad \text{U} \\ 1 \quad 2 \\ \underline{6 \quad 9} + \end{array}$$

Step 2: add the units column - carry the tens over.

$$\begin{array}{r} \text{T} \quad \text{U} \\ 1 \quad 2 \\ \underline{6 \quad 9} + \\ \underline{\quad 1} \\ 1 \end{array}$$

$2 + 9 = 11$

- So place a 1 in the units column
- then carry the 10 over.

Step 3: add the tens column including the carry over.

$$\begin{array}{r} \text{T} \quad \text{U} \\ 1 \quad 2 \\ \underline{6 \quad 9} + \\ \underline{8 \quad 1} \\ 1 \end{array}$$

Add the 1 and the 6 in the tens column.

$1 + 6 = 7$

then add the carry over on so

$7 + 1 = 8$

Remember this number is in the tens column so 8 stands for 80.

Visual SUBTRACTION

$$4 - 1 = ?$$

Find something you can easily pick up - like buttons.

Pick up four buttons and put them in a row.



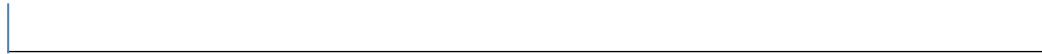
Then take one button away and count the buttons remaining.



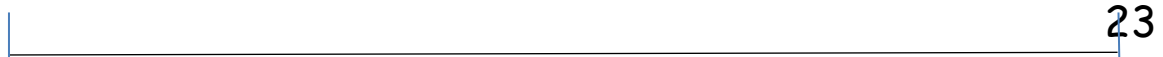
SUBTRACTION using a number line.

$$23 - 12 = ?$$

- 1) Draw a line:



- 2) Put the biggest number at the end of the line.



- 3) Make jumps on the number line according to the number you are subtracting. Work right to left.



Again, like with addition, the size of the steps are your choice.

Another example: $87 - 25 =$

Step one - take away the units (5)



Step two - then take away the tens (20) *this could be done in two steps of 10.*



Or... we could take the tens away first and then the units. T



SUBTRACTION : Partitioning

$$22 - 11 =$$

First we must partition the numbers into **tens** and **units**.

$$22 = 20 \text{ and } 2$$

$$11 = 10 \text{ and } 1$$

Then subtract one ten from the other and then do the same with the units.

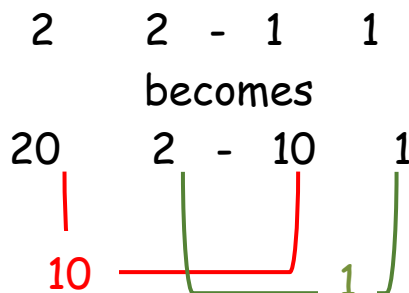
$$20 - 10 = 10$$

$$2 - 1 = 1$$

and then we add these results for our answer $10 + 1 = 11$
we have to add here because we have to put our tens and units back together again after partitioning them.

.....

OR... in a more visual way this could be done as below.



Again to get our answer we have to add the partitioned results. So $10 + 1 = 11$

Another way to partition with bigger numbers is...

$$672 - 321 =$$

$$\begin{array}{r} 600 + 70 + 2 \\ - 300 + 20 + 1 \\ \hline 300 + 50 + 1 = 351 \end{array}$$

N.B. There are problems with these methods when there is a need to borrow!

SUBTRACTION : Standard Columns

$22 - 11 =$

Step one... write the calculation in columns.

$$\begin{array}{r} \text{TU} \\ 22 \\ - 11 \\ \hline \end{array}$$

KEY:

U = units

T = tens

Step two... subtract the **units** column

$$\begin{array}{r} \text{TU} \\ 22 \\ - 11 \\ \hline \underline{1} \end{array}$$

Step three... subtract the **tens** column

$$\begin{array}{r} \text{TU} \\ 22 \\ - 11 \\ \hline \underline{11} \end{array}$$

Subtraction with borrowing...

Step 1: write the calculation.

$$\begin{array}{r} \text{T} \quad \text{U} \\ 6 \quad 2 \\ - \underline{1 \quad 9} \end{array}$$

KEY:

U = units

T = tens

Step 2: subtract the units column - borrowing from the tens column.

$$\begin{array}{r} \text{T} \quad \text{U} \\ 5 \quad \cancel{6} \mid 2 \\ - \underline{1 \quad 9} \\ \hline \quad \quad 3 \end{array}$$

2 - 9 doesn't work so we have to borrow one of the tens. We cross out the 6 and replace it a 5 (standing for 5 tens). Then we carry over one ten and then our new calculation is 12 - 9 which equals 3

Step 3: subtract the tens column.

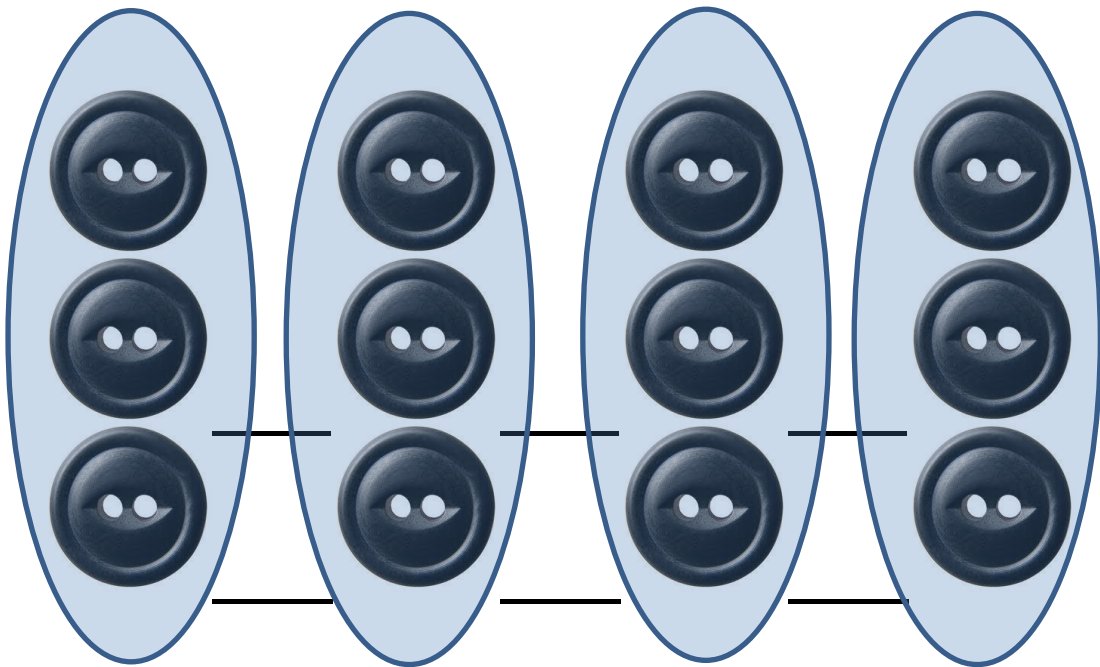
$$\begin{array}{r} \text{T} \quad \text{U} \\ 5 \quad \cancel{6} \mid 2 \\ - \underline{1 \quad 9} \\ \hline 4 \quad 3 \end{array}$$

As we borrowed a ten we only have 5 tens left. So, 5 - 1 = 4

MULTIPLICATION : visual

$$4 \times 3 = ?$$

Put resources in appropriate piles and then count them.



If we count them up we can see that 4 lots of 3 or $4 \times 3 = 12$

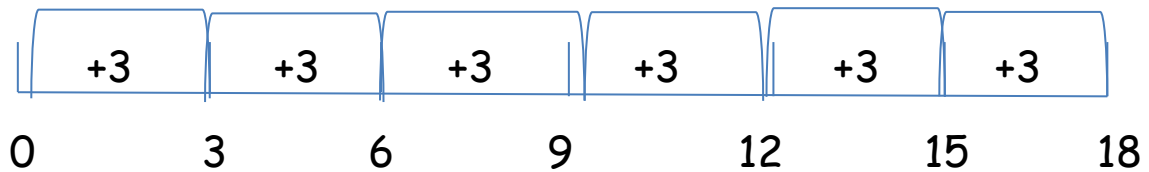
Below is a different set up for the same calculation:



MULTIPLICATION : number line

$$6 \times 3 =$$

Children start by using a number line to add the groups:



There are 6 jumps on the number line, each one adding 3 and giving a total of 18.

MULTIPLICATION : grid method

This method is effective for bigger numbers. As for addition, children partition any numbers with 2 or more digits:

$$15 \times 12$$

Draw a grid 3 x 3

Place the x (standing for multiply) in the top left hand square. Then write the digits in the columns by partitioning the tens and units in each column / row.

12 is written as 10 and 2. 15 is written as 10 and 5.

x	10	5
10		
2		

Begin to fill in the grid by following the row by column calculations.

Step 1

x	10	5
10	100	
2		

Step 2

x	10	5
10	100	
2	20	

Step 3

x	10	5
10	100	50
2	20	

Step 4

x	10	5
10	100	50
2	20	10

Step 5

Add up each of the results -

$$100 + 50 + 20 + 10 = 180$$

So therefore $15 \times 12 = 180$

Now for HTU times TU

With multiplication like this it is important to note that being able to solve 30×3 or similar calculations, relies on children using what they know about $3 \times 3 = 9$,

They should remember that 30 is ten times bigger than 3. Therefore the answer will be 10 times bigger so

$$30 \times 3 = 90$$

$$234 \times 23 =$$

Draw a grid 4 x 3

x			

Write the digits in the columns by partitioning the hundreds, tens and units in each column / row.

x	200	30	4
20			
3			

Begin to fill in the grid by following the row by column calculations.

Step 1 -

x	200	30	4
20	4000		
3			

Step 2-

x	200	30	4
20	4000		
3	600		

Step 3-

x	200	30	4
20	4000	600	
3	600		

Step 4-

x	200	30	4
20	4000	600	
3	600	90	

Step 5 -

x	200	30	4
20	4000	600	80
3	600	90	

Step 6 -

x	200	30	4
20	4000	600	80
3	600	90	12

Step 7 - Now the grid is full add the results...

(note if there are 6 result boxes then there will be 6 numbers to add in the column addition.)

$$\begin{array}{r} 4000 \\ 600 \\ 600 \\ 90 \\ 80 \\ + 12 \\ \hline 5382 \\ 11 \end{array}$$

So $234 \times 23 = 5382$ 😊

N.B For different calculations the look may be different but the procedure is the same.

$$124 \times 5$$

x	100	20	4
5			

NOTE:

We decide the size of the grid dependent on the size of the calculation.

$$\text{TU} \times \text{TU} = 3 \times 3$$

$$\text{HTU} \times \text{TU} = 4 \times 3$$

$$\text{HTU} \times \text{HTU} = 4 \times 4$$

$$\text{HTU} \times \text{U} = 4 \times 2$$

From looking at the examples above you can see that basically you need to add a row or column for each number when partitioned.

MULTIPLICATION : Partitioning

Step one... write the calculation.

$$62 \times 3 = \begin{array}{r} 62 \\ \times 3 \\ \hline \end{array}$$

Step two... multiply the units.

$$\begin{array}{r} \text{TU} \\ 62 \\ \times 3 \\ \hline 6 \end{array}$$

The calculation here is
 $2 \times 3 = 6$

Step three... multiply the tens column

$$\begin{array}{r} \text{TU} \\ 62 \\ \times 3 \\ \hline 6 \\ 180 \end{array}$$

The calculation here is
 $60 \times 3 = 180$

Step four... add the two answers together

$$\begin{array}{r} \text{TU} \\ 62 \\ \times 3 \\ \hline 6 \\ +180 \\ \hline 186 \end{array}$$

The calculation here is
 $6 + 180 = 186$

For TU x TU

$62 \times 43 =$

$$\begin{array}{r} 62 \\ \times 43 \\ \hline \end{array}$$

Step 1 : multiply the units by units column

$$\begin{array}{r} 62 \\ \times 43 \\ \hline \end{array}$$

The calculation here is
 $2 \times 3 = 6$

Step 2: Then multiply the ten by the unit

$$\begin{array}{r} 62 \\ \times 43 \\ \hline 6 \\ 180 \end{array}$$

The calculation here is
 $60 \times 3 = 180$

Step 3: Now multiply the unit by the tens column

$$\begin{array}{r} 62 \\ \times 43 \\ \hline 6 \\ 180 \\ \hline 80 \end{array}$$

The calculation here is
 $40 \times 2 = 80$

Step 3b: now multiply the ten column by the ten column.

$$\begin{array}{r} 62 \\ \times 43 \\ \hline 6 \\ 180 \\ 80 \\ \hline 2400 \end{array}$$

The calculation here is
 $60 \times 40 = 2400$

Step 5: Finally... add the four results together

$$\begin{array}{r} 62 \\ \times 43 \\ \hline 6 \\ 180 \\ 80 \\ 2400 + \\ \hline 2666 \\ 1 \end{array}$$

MULTIPLICATION : Standard Columns

Step one... write the calculation.

$$62 \times 3 = \begin{array}{r} 62 \\ \underline{\times 3} \end{array}$$

Step two... multiply the units.

$$\begin{array}{r} 62 \\ \underline{\times 3} \\ \underline{6} \end{array}$$

Step three... multiply the tens column

$$\begin{array}{r} 62 \\ \underline{\times 3} \\ \underline{186} \end{array}$$

For TU x TU

$62 \times 43 =$

$$\begin{array}{r} 62 \\ \times 43 \\ \hline \end{array}$$

Step 1: As before multiply by the units column

$$\begin{array}{r} 62 \\ \times 43 \\ \hline 6 \end{array}$$

Then...

$$\begin{array}{r} 62 \\ \times 43 \\ \hline 186 \end{array}$$

Step 2: Now multiply by the tens column but because we are multiplying by ten we must place a zero in the units column to show there are no units.

$$\begin{array}{r} 62 \\ \times 43 \\ \hline 186 \\ \underline{0} \end{array}$$

$$\begin{array}{r}
 62 \\
 \times 43 \\
 \hline
 186 \\
 80 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 62 \\
 \times 43 \\
 \hline
 186 \\
 2480 \\
 \hline
 \end{array}$$

Step 3: finally add the two results together

$$\begin{array}{r}
 62 \\
 \times 43 \\
 \hline
 186 \\
 2480+ \\
 \hline
 2666 \\
 1
 \end{array}$$

DIVISION : visual

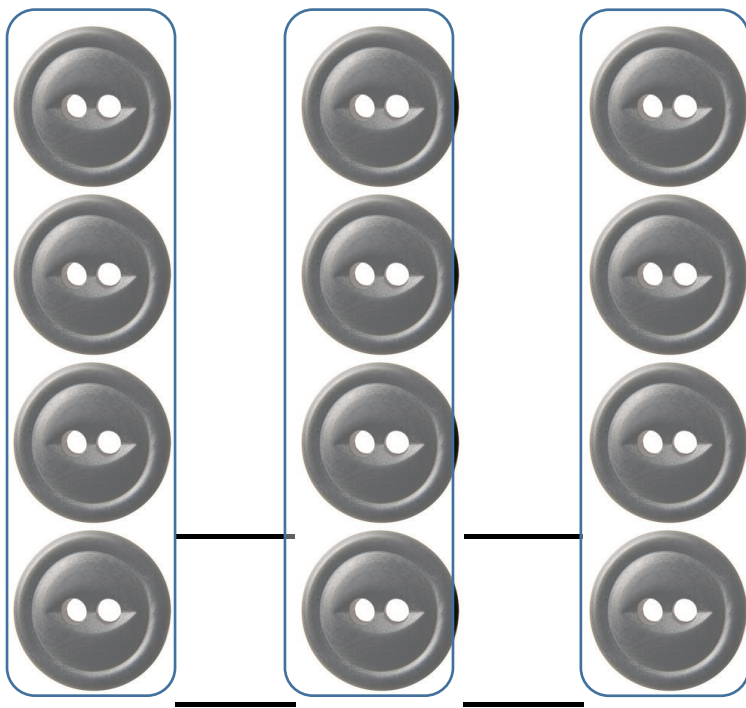
$$12 \div 3 =$$

For this calculation you need 12 buttons or something similar. Share them into three separate piles.

Here we have three lines/ piles of four buttons:

12 shared into 3 piles = 4 in each pile

$$12 \div 3 = 4$$



This is another way to see the same thing.



DIVISION : Chunking

(we need to chunk in no more than two or three steps.)

Step one... write the calculation.

$$66 \div 3 =$$

Step two... To help look at the number to divide by (in this case it is 3) and then note down a few multiplication tables to help.

$$1 \times 3 = 3$$

$$2 \times 3 = 6$$

$$3 \times 3 = 9$$

$$4 \times 3 = 12$$

$$5 \times 3 = 15$$

We can use this information to help us calculate the same number ten times bigger so we can get closer to the number.

$$10 \times 3 = 30$$

$$20 \times 3 = 60$$

$$30 \times 3 = 90$$

Step three... draw a bus stop and insert the numbers. The number we are dividing by is called the divisor and this is inserted on the left.

$$\begin{array}{r} \leftarrow \hspace{1.5cm} \hspace{0.5cm} \rightarrow \\ 3 \overline{) 66} \end{array}$$

Step four we need to get as close as we can to the number we are dividing into but not over it. Using the multiplication table we can see that $20 \times 3 = 60$ is close, so we use this and record it as below. We show the calculation in brackets, underlining the number of 3s used so that we can work out how many chunks there are later.

$$\begin{array}{r} 3 \overline{) 66} \\ - 60 \quad (\underline{20} \times 3) \\ \hline \end{array}$$

Step five... now calculate $66 - 60$ which equals 6, then calculate how many 3s into the remaining 6 and record in brackets - underlining the number of 3s.

$$\begin{array}{r} 3 \overline{) 66} \\ - 60 \quad (\underline{20} \times 3) \\ \hline 6 \\ \underline{6} \quad (\underline{2} \times 3) \\ \hline \end{array}$$

0 (this indicates that there are no remainders. Remainders are what is left over - see glossary)

Step six... look at the numbers underlined in the brackets - these have to be added up $20 + 2 = 22$ and that gives you the answer.

$$\begin{array}{r} 22 \\ 3 \overline{) 66} \\ - 60 \quad (\underline{20} \times 3) \\ \hline 6 \\ \underline{6} \quad (\underline{2} \times 3) \\ \hline \end{array}$$

0 (this indicates that there are no remainders.)

Here is another example with a remainder:

Step one... write the calculation and the multiplication facts to help.

$$2676 \div 43$$

$1 \times 43 = 43$	$10 \times 43 = 430$
$2 \times 43 = 86$	$20 \times 43 = 860$
$5 \times 43 = 215$	$50 \times 43 = 2150$

Step two... draw a bus stop and insert the numbers!

$$43 \overline{) 2676}$$

Step three... calculate how many 43s go into the number 2676 and write in brackets next to the answer underlining how many times you have multiplied 43. Use the table written earlier to help. As in the earlier example it is important to get as close as you can to the number without going over it.

$$43 \overline{) 2676} \\ 2150 \text{ (} \underline{50} \times 43 \text{)}$$

Step four... now calculate 2676- 2150.

$$\begin{array}{r} 43 \overline{) 2676} \\ \underline{-2150} \quad (\underline{50} \times 43) \\ 0526 \end{array}$$

Step five... Now, you can calculate how many 43s go into the remaining 526. Again make a note in brackets and underline the amount you times 43 by.

$$\begin{array}{r} 43 \overline{) 2676} \\ \underline{-2150} \quad (\underline{50} \times 43) \\ 0526 \\ \underline{0430} \quad (\underline{10} \times 43) \end{array}$$

Step six... now calculate 526 - 430

$$\begin{array}{r} 43 \overline{) 2676} \\ \underline{-2150} \quad (\underline{50} \times 43) \\ 0526 \\ \underline{0430} \quad (\underline{10} \times 43) \\ 96 \end{array}$$

Step seven... Now, you can calculate how many 43s go into the remaining 96. Again make a note in brackets and underline the amount you times 43 by.

$$\begin{array}{r}
 43 \overline{) 2676} \\
 \underline{-2150} \quad (50 \times 43) \\
 0526 \\
 \underline{0430} \quad (10 \times 43) \\
 96 \\
 \underline{86} \quad (2 \times 43)
 \end{array}$$

Step eight... now calculate $96 - 86$

$$\begin{array}{r}
 43 \overline{) 2676} \\
 \underline{-2150} \quad (50 \times 43) \\
 0526 \\
 \underline{0430} \quad (10 \times 43) \\
 96 \\
 \underline{86} \quad (2 \times 43) \\
 10
 \end{array}$$

As 43 will not go into 10, this is your remainder!

So we now add up all the underlined numbers in brackets -

$$50 + 10 + 2 = 62$$

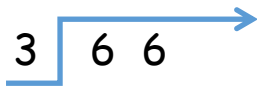
Therefore $2676 \div 43 = 62$ remainder 10

DIVISION : Standard

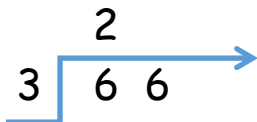
Step one... write the calculation.

$$66 \div 3$$

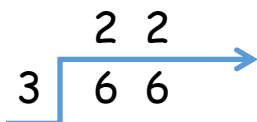
Step two... draw a bus stop and insert the numbers!


$$\begin{array}{r} 3 \overline{) 66} \end{array}$$

Step three... calculate how many divisors go into the tens


$$\begin{array}{r} 2 \\ 3 \overline{) 66} \end{array}$$

Step four... calculate how many divisors go into the units


$$\begin{array}{r} 22 \\ 3 \overline{) 66} \end{array}$$

Standard Division with a remainder...

Step one... write the calculation.

$$68 \div 3$$

Step two... draw a bus stop and insert the numbers.

$$\begin{array}{r} 3 \overline{) 68} \end{array}$$

Step three... calculate how many divisors go into the tens

$$\begin{array}{r} 2 \\ 3 \overline{) 68} \end{array}$$

Step four... calculate how many divisors go into the units and then what ever is left over is called the remainder (r).

$$\begin{array}{r} 22 \text{ r } 2 \\ 3 \overline{) 68} \end{array}$$

Standard Division with a carry over...

Step one... write the calculation.

$$76 \div 3$$

Step two... draw a bus stop and insert the numbers!

$$\begin{array}{r} 3 \overline{) 76} \end{array}$$

Step three... calculate how many divisors go into the tens and then carry over anything left over - place this left over next to the next number to be calculated.

$$\begin{array}{r} 2 \\ 3 \overline{) 76} \end{array}$$

Step four... calculate how many divisors go into the units including the carry over. If there is a remainder, record it after a small 'r'.

$$\begin{array}{r} 25 \text{ r } 1 \\ 3 \overline{) 76} \end{array}$$

DIVISION : Standard long

Step one... write the calculation.

$$66 \div 3$$

Step two... draw a bus stop and insert the numbers!

$$\begin{array}{r} 3 \overline{) 66} \end{array}$$

Step three... calculate how many divisors go into the tens and then write the answer at the top but also show the answer below and carry down the units

$$\begin{array}{r} 2 \\ 3 \overline{) 66} \\ - \underline{6} \downarrow \\ 06 \end{array}$$

Step four... calculate how many divisors go into the units but also show the answer and the calculation below

$$\begin{array}{r} 22 \\ 3 \overline{) 66} \\ \underline{- 6} \\ 06 \\ \underline{- 6} \\ \underline{0} \end{array} \text{ (this indicates that there are no remainders.)}$$

Here is another example:

Step one... write the calculation.

$$2666 \div 43$$

Step two... draw a bus stop and insert the numbers!

$$43 \overline{) 2666}$$

Step three...calculate how many 43s go into 2. Write the answer 0 at the top but also show the answer below and carry down the numbers.

$$\begin{array}{r} 0 \\ 43 \overline{) 2666} \\ - \quad 0 \downarrow \\ \hline 26 \end{array}$$

Step four... continue this with each number you come to. Here we have how many 43s go in 26. So we record 0 again and then carry down the next 6 to make 266.

$$\begin{array}{r}
 43 \overline{) 2666} \\
 \underline{- 0} \\
 26 \\
 \underline{ 0} \\
 266
 \end{array}$$

Step five - Here we have to work out how many 43s go into 266. So we record 6 and the answer 258. We then take 258 from 266 and are left with 8.

$$\begin{array}{r}
 43 \overline{) 2666} \\
 \underline{- 0} \\
 26 \\
 \underline{ 0} \\
 266 \\
 \underline{- 258} \quad (43 \times 6) \\
 008
 \end{array}$$

Step six... finally... we carry down the final 6. Now, we have to work out how many 43s go into 86. So we record 2 and the answer 86. We then take 86 from 86 and are left with 0 (*so there are no remainders*).

$$\begin{array}{r}
 006\mathbf{2} \\
 43 \overline{) 2666} \\
 \underline{- 0} \\
 26 \\
 \underline{- 0} \\
 266 \\
 \underline{- 258} \\
 0086 \\
 \underline{0086} - (43 \times 2) \\
 \hline
 0
 \end{array}$$

N.B. Unlike chunking, with this method we have to find the biggest number that the divisor will go into.

GLOSSARY

- Addition**
- To join two or more numbers / quantities to get one number (called the sum or the total).
 - The opposite of subtraction.
- Dividend**
- The number being divided.
- Division**
- Sharing or grouping a number into equal parts.
 - The opposite / inverse of multiplication.
- Divisor**
- A number that divides into the dividend.
- Multiplication**
- The operation used to put numbers of the same size together.
 - Repeated addition. e.g. $5 \times 3 = 5 + 5 + 5$
 - The opposite of division.
- Partitioning**
- Breaking down a number into smaller numbers to help solve problems e.g. if the number is 254
We break it down into:
Hundreds - 2 (200)
Tens - 5 (50)
Units - 4 (4)
- Place value**
- Place value describes the value of the place of a digit in a number. For example, the 4 in 47 is in the tens place, giving it a total value of 4×10 , or 40.
- Remainder**
- The amount left over after division.
- Subtraction**
- To take one number / quantity away from another.
 - The opposite of addition

Sum

- The total or whole amount.
- The result of adding.

Words which could be used in place of each number operation

Add	sum, plus, together, added to, total of, combine, both
Subtract	difference, minus, take away, fewer than, less than, decreased by, reduce
Multiply	product of, multiplied by, times, lots of, groups of
Divide	quotient of, divided by, into, out of, cut up, separated , share equally, split, group